Comparing The Solutions for Vehicle Routing Problem with Uncertain Travel Times by Robustness Approach

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IJLT International Journal of Logistics and Transport

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ABSTRACT

The main objective of this paper is to find a robust solution of a vehicle routing problem (VRP) that minimizes (over all solutions) the maximum (over all scenarios) of a performance measure. This VRP model reflects the intrinsic difficulties in estimating travel times exactly in reality such as traffic conditions, accidents, traffic jams or weather conditions. Accordingly, the critical element of this paper is the uncertainty model by using the scenario-based approach. The three well-known VRP techniques are used to find the best solution for each scenario. Method I is the simultaneous route and cluster by savings algorithm, method II is the route first-cluster second and method III is the cluster first-route second. This paper presents robustness criteria, namely, the robust deviation criterion, the sum square error robustness criterion and the robust risk criterion. The experiments set a VRP with 3 inducement scenarios (Scenario I), as the optimistic case, the most likely case and the worst case of traffic conditions. One hundred realization scenarios (Scenario S) occur in the experiments. The results clearly show that method I has higher performance than the other two methods. The best solution from the most likely scenario is the minimax solution for the robust deviation criterion and the sum square error robustness criterion. The worst case scenario is appropriate to the robust risk criterion.

KEYWORDS
Robustness Criteria, Vehicle Routing Problem, Uncertain Travel Times, Scenario-Based Approach

INTRODUCTION

Vehicle routing problem (VRP) is operational decision-making for the delivery of goods from a depot (or multiple depots) to customers by a fleet of vehicles. The objective of VRP is to find the optimal routes for delivery vehicles on the road network that minimize the number of vehicles required as well as the distance traveled under constraints on capacity, travel time, and customer demand. This type of problem becomes complex when some parameters are uncertain. Therefore, there is a need to develop routing and scheduling tools that account directly for the uncertainty. Recently, researchers have begun to study such problems and to develop approaches for finding robust solutions which have the best worst-case performance over a set of possible scenarios. Kouvelis and Yu (1977) discuss approaches for handling uncertainty and they review robust discrete optimization problems and a wide variety of their applications. This paper focuses on uncertain travel times due to traffic and travel conditions.

The VRP techniques include the simultaneous route and cluster, the route first-cluster second and the cluster first-route second. These algorithms are the heuristic and meta-heuristic techniques for solving "the minimax optimization problems", which is the same as minimizing (over all solutions) the maximum (over all scenarios) performance. Its objective is to find a robust route that minimizes the worst-case value over all data sets of uncertainty. The traditional approaches for handling uncertainty in decision-making have been divided conventionally into three categories: the deterministic optimization approach, the stochastic optimization approach and the robust optimization approach. A major weakness of the deterministic approach is its inability to recognize the presence of plausible data instances other than the "most likely" one used to generate the "optimal" decision. The stochastic optimization approach does recognize the presence of multiple data instances that might be potentially realized in the future. However, the failure of both stochastic and deterministic optimization approaches is their inability to recognize every decision. There is a whole distribution of outcomes depending on which data scenario is actually realized. Thus any approach that evaluates decisions using only a single data scenario or the best solution has been selected in the expected outcome. In this research aspect, decision makers in a decision environment with significant uncertainty, want a robust decision, one that performs well across all scenarios. It hedges against the worst of all possible scenarios while the probability of data is unknown exactly. We use a scenario-based approach, structuring uncertainty so that it is part of the robust approach, requiring executives to participate in the generation and evaluation processes for all scenarios.

The rest of this paper is organized as follows: section 2, the literature review focuses on literature which is relevant to the vehicle routing problem with uncertain travel times. Section 3, the problem formulation and the method explain how the problem is formulated and solved. Section 4 is the results and discussion of the experimental results. Finally, conclusions and future work are explained in the last section.

LITERATURE REVIEW

The field of decision-making under uncertainty was pioneered in the 1950s by Dantzig (1955), and Charnes and Cooper (1959), who set the foundation for stochastic programming and optimization under probabilistic constraints. Stochastic programming has established itself as a powerful modeling tool when an accurate probabilistic description of the randomness is available. However, in many real-life applications the decision-maker does not have this information (Montemanni *et al.*, 2007). Estimating travel times exactly is typically a difficult task, since they depend on many factors that are difficult to predict, such as traffic conditions, accidents, traffic jams or weather conditions.

Most VRP models assume that travel times are a deterministic approach. This approach either completely ignores uncertainty or uses historical data and trends to forecast the future. The major weakness of the deterministic approach is that it does not capture the reality of problems. In the world of uncertainty, the stochastic approach does recognize the presence of multiple data instances that might be potentially realized in the future. It is better in representing the reality of the problems. However, before feeding the data instances to the decision model, they might be randomly drawn from the assumed probability distribution. The failure of both the stochastic and the deterministic approaches is their inability to recognize for all environments. The decision makers are reasonably more interested in hedging against the risk of poor system performance for some realizations of data scenarios than in optimizing expected system performance over all potential scenarios. This paper presents the decision environments with significant uncertainty as a robust decision. The robustness approach is decision-making due to uncertainty, in which the uncertainty model is not stochastic or the probability is unknown parameters (Bertsimas et al., 2007).

In the mid-1990s, research teams led by Ben-Tal and Nemirovski (1998; 1999; 2000) and El-Ghaoui and Lebret (1997) addressed the uncertain parameters belonging to ellipsoidal sets, which remove the most unlikely outcomes of consideration and yield tractable mathematical programming problems. In line with these authors' terminology, optimization for the worst-case value of parameters within a set becomes known as "robust discrete optimization". The robust decision under the worst-case performance is the best solution under the robust approach but it leads to being overly conservative and high cost. The scenario-based approach is an alternative method to avoid the overly conservative solution. The scenario-planning process is challenging when implemented in large organizations. Herrmann (1999) applies the scenario-based approach to model the uncertainty and uses a two-space genetic algorithm to find the optimal makespan for the robust parallel machine scheduling problem. Montemanni *et al.* (2007) present a new extension to the traveling salesman problem (TSP), where the travel times are specified as a range of possible values. They apply the robust deviation criterion and the exact methods to solve optimization. These exact methods are available for small-scale problems but they might not be feasible for large-scale problems.

This paper proposes the scenario-based approach to model the uncertainty of travel times. The minimax optimization problem is the main objective of this paper. It finds a robust solution under the robust criteria by using VRP techniques. In the next section, we describe the method to solve the VRP with uncertain travel times where a travel time is defined as a finite set interval.

METHODS

1. Notation

K : Number of vehicles
N : Number of customers and depots
C_i : Customer at node i for i = 0, 1, 2, 3,…,N
C_1 : Depot
t_{ij} : Travel times between customer i to customer j
u_i : Arbitrary number for customer i
d_i : Demand for customer i
q_k : Capacity of vehicle

2. Definition

This paper proposes the scenario-based approach with three robustness criteria as the following definitions.

Definition 1: A scenario S is realization of the arc travel times, i.e. t_{ij} is the travel time for each pair of customers i to j. The interval of the travel times is defined as $[l_{ij}, u_{ij}]$ where l_{ij} is minimum travel time and u_{ij} is maximum travel time. The scenario S is randomly generated from a $[l_{ij}, u_{ij}]$ and a matrix of the travel times costs constructed.

Definition 2: A scenario I is inducement of significant events that might occur in the present and the future.

Definition 3: The robust deviation criterion is defined as the one that exhibits the best worst-case deviation among all feasible decisions over all realizable input data scenarios, i.e.,

$$dev(X,S) = \left| \sum_{(i,j)\in X} t_{ij}{}^{I} - \sum_{(i,j)\in X} t_{ij}{}^{S} \right| \qquad (1)$$

Definition 4: The sum square error robustness criterion is the sum square difference between the total travel times $\sum t_{ij}$ in scenario I and scenario S.

$$sse(X,S) = \left(\sum_{(i,j)\in X} t_{ij}{}^{I} - \sum_{(i,j)\in X} t_{ij}{}^{S} \right)^2 \qquad (2)$$

Definition 5: The robust risk criterion occurs when the total travel times $\sum t_{ij}$ in scenario S are more than scenario I.

$$risk(X,S) = \sum_{(i,j)\in X} t_{ij}{}^{I} - \sum_{(i,j)\in X} t_{ij}{}^{S} \qquad (3)$$

$$if \sum_{(i,j)\in X} t_{ij}{}^{I} < \sum_{(i,j)\in X} t_{ij}{}^{S} \rightarrow risk(X,S) = \left| \sum_{(i,j)\in X} t_{ij}{}^{I} - \sum_{(i,j)\in X} t_{ij}{}^{S} \right|, \text{ else } risk(X,S) = 0$$

The remainder of this paper refers to the robust deviation criteria of tour, x on scenario s as $dev(X,S)$, the sum square error robustness as $sse(X,S)$ and the robust risk as $risk(X,S)$, respectively.

3. Mathematical Formulation

This section introduces a general deterministic VRP and a robust VRP. The decision variables of this problem are $X_{ijk} = 1$ if edge $\{i, j\}$ for vehicle number k is on the tour and 0 for otherwise. The deterministic VRP is a counterpart of the robustness approach. The objective of individual deterministic VRP is to minimize the number of vehicles that can serve all customers' demand and find the best route with minimum travel times. The mathematical model of VRP is as follows;

Deterministic VRP :

Objective functions;

$$Z_1 = MinK \qquad (4)$$

$$Z_2 = Min\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k-1}^{K-1} t_{ij} X_{ijk} \qquad t_{ij} \in [l_{ij}, u_{ij}] \;\; k = 1,...,K \qquad (5)$$

Constraints listed below;

$$\sum_{k=0}^{K-1}\sum_{j=1}^{N} X_{ijk} = K \qquad i = 0 \qquad (6)$$

$$\sum_{k=0}^{K-1}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_{ijk} = 1 \qquad i = 1,2,3,...,N \qquad (7)$$

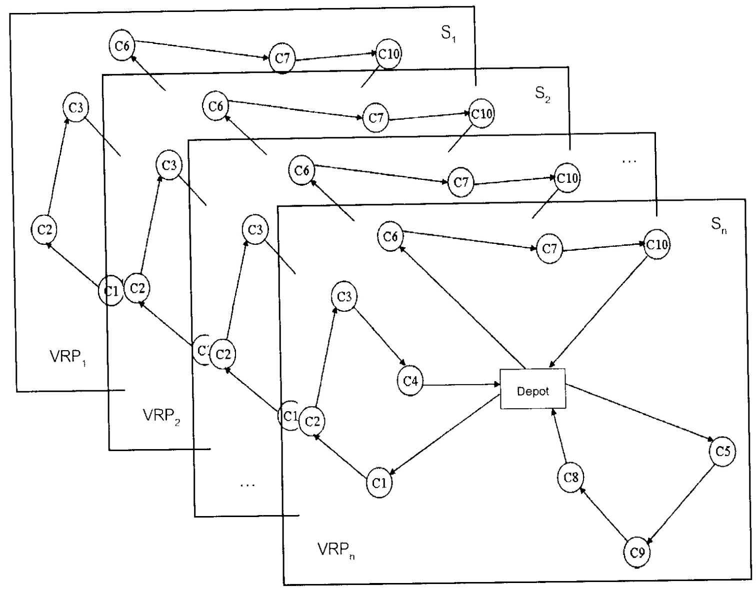
$$\sum_{\substack{i=0 \\ i\neq h}}^{N} X_{ihk} - \sum_{\substack{j=0 \\ j\neq h}}^{N} X_{ijk} = 0 \qquad \forall h \in [1, N]; k \in [0, K-1] \qquad (8)$$

$$u_i - u_j + Nx_{ij} \leq N - 1 \qquad i = 1,2,3,...,N; j = 1,2,3,...,N; i \neq j \qquad (9)$$

$$\sum_{i=0}^{N}\sum_{\substack{j=0 \\ j\neq i}}^{N} d_i X_{ijk} \leq q_k \qquad \forall k \in [0, K-1] \qquad (10)$$

The constraint in equation (6) specifies that the number of tours to service must not exceed the available quantity of vehicles. The number of vehicles to service is stated by the total number of vehicles flowing from and then turning back to the depot. Constraint (7) states that each demand node must be visited only once. Constraint (8) requires that all vehicles that flow into a demand point must flow out of it. Constraint (9) prevents sub-tour occurring in the solution. Constraint (10) illustrates the capacity of the vehicles.

The robust VRP is a several deterministic VRP due to the number of scenarios. Figure 1 illustrates a VRP with n scenarios. The objective is finding a minimax total travel times of scenario inducement that represent the robust VRP solution.



The robust VRP can be formulated as follows. Let X be the set of all solutions. Let S be the set of all possible scenarios. The performance of the solution is $x \in X$ and the scenario, $s \in S$ is $g_s(X)$. The problem is to find the solution that has the best worst-case performance, which is the same as minimizing (over all solutions) the maximum (over all scenarios) performance.

Robust VRP :

Objective functions;

$$Z = \min\{h_x(S) \mid g_s(X) \le h_x(S), s \in S; X \in \cap_{s \in S} F_s\} \tag{11}$$

Constraints functions;

$$g_s(X) = \begin{cases} dev(X,S) \\ sse(X,S) \\ risk(X,S) \end{cases} \tag{12}$$

The objective function in equation (11) is made explicit as equation (12). The other constraints of this problem are the same as the original VRP for deterministic optimization.

4. Scenario-Based Approach

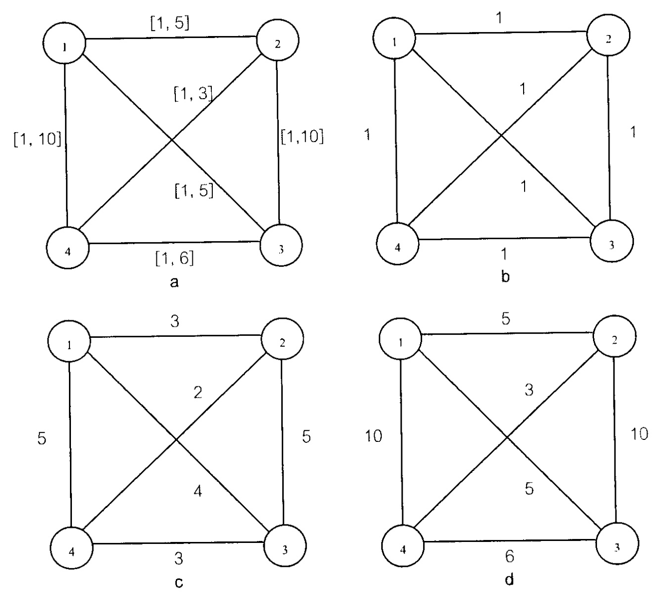
This paper uses the VRP techniques based on the scenario-based approach to solve the robust solutions under interval uncertainty of travel times. The scenario-based approach represents the uncertainty through a finite set of uncertainties. A scenario model can be seen as a snapshot representing the transportation network situation as a path of any possible edge cost configuration.

FIGURE 2A
SIMPLE EXAMPLE VRP WITH INTERVAL TRAVEL TIMES

FIGURE 2B
SCENARIO 1: THE OPTIMISTIC CASE

FIGURE 2C
SCENARIO 2: THE MOST LIKELY CASE

FIGURE 2D
SCENARIO 3: THE WORST CASE



5. Heuristic Methods

This paper implemented the well-known VRP techniques to find the best route for each scenario: the simultaneous route and cluster method, the route first-cluster second method and the cluster first-route second method. These three techniques use the heuristic algorithm as shown below.

5.1 The simultaneous route and cluster method (method I)

This method applies the savings algorithm (Clark and Wright, 1964) and improves the initial solutions by 2-opt exchanges method. The algorithm is shown below.
Step 1: Calculate the savings $s(i,j) = d(D,i) + d(D,j) - d(i,j)$ for every pair (i,j) of demand nodes.

Step 2: Rank the savings $s(i,j)$ and list them in descending order of magnitude. This creates the "savings list". Process the savings list beginning with the topmost entry in the list (the largest $s(i,j)$).

Step 3: For the savings $s(i, j)$ under consideration, include link (i, j) in a route if no route constraints will be violated through the inclusion of (i, j) in a route, and if:

a. Either, neither i nor j have already been assigned to a route, in which case a new route is initiated including both i and j.
b. Or, exactly one of the two points (i or j) has already been included in an existing route and that point is not interior to that route (a point is interior to a route if it is not adjacent to the depot D in the order of traversal of points), in which case the link (i, j) is added to that same route.
c. Or, both i and j have already been included in two different existing routes and neither point is interior to its route, in which case the two routes are merged.

Step 4: If the savings list $s(i, j)$ has not been exhausted, return to Step 3, processing the next entry in the list, otherwise, stop the solution to the VRP consists of the routes created during Step 3.
(Any points that have not been assigned to a route during Step 3 must each be served by a vehicle route that begins at the depot D visits the unassigned point and returns to D.)

Step 5: Select the link (i, j) of initial solution and then swap it.

Step 6: Check the fitness function, if it is improving, construct a new route of the tour, else the existing route must not be changed.

Step 7: Stop when the solution is not better or near optimum.

5.2 The route first-cluster second method (method II)

Method II constructs the route first by using the genetic algorithm (GA) and then applies sweeping algorithm for seeking the cluster of cities.

Step 1: Adapt GA for TSP (Joseph, 2008) to initialize the route construction.

Step 2: Assume the polar coordinates are available for all points of the cities to be visited by the vehicles.

Step 3: Start the origin in the coordinate system at the depot node.

Step 4: Order in terms of increasing angle by sweeping (clockwise or counterclockwise) a ray initially drawn from the depot to some arbitrary point known as the seed point.

Step 5: Routes are then drawn up by adding demand points to a route as these demand points are swept beginning at the seed (whose angle can be set to 0), points are included in a route as they are swept until the load capacity of a vehicle precludes addition of the next point swept to the current route.

Step 6: That point then becomes the seed for the next route and the process is completed when all points have been swept (i.e., included in a route).

5.3 The cluster first-route second method (method III)

Method III is a conversion of method II. First step becomes the sweeping algorithm for the cluster of cities and then re-arrangement of the initial route by using GA.

6. Computational Studies

The VRP with uncertain travel times can be transformed to the deterministic VRP represented by scenario model. An example of this problem shows three traffic conditions. There are three inducement scenarios, thus the optimistic case, the most likely case and the worst case. Each case has a vehicle's velocity (kilometer per hour), v_{ij} = 100, 80 and 40, respectively. The travel times between any pair of cities is calculated by Euclidean distances. Table 3.1 is a simple VRP with 10 cities of depot and customers. The vehicle capacity is equal to 800 units. Every day the vehicles must start at a depot and return to transfer their load after visiting all customers in their tour. The objective of this problem is to find a robust route with the minimax total travel times according to the cost function associated with the chosen notions of the robustness criteria.

TABLE 3.1
A SIMPLE VRP

Cities	x	y	Demand
1	20	30	0
2	15	19	100
3	19	21	250
4	10	20	120
5	25	32	200
6	16	50	170
7	25	44	100
8	12	64	300
9	23	73	180
10	20	65	50
	Total demand		1470

RESULTS AND DISCUSSION

From section 3.6, there are 9 times of computational running for all of combinatorial, 3 inducement scenarios and 3 heuristic methods. Table 4.1 shows an example of the computational result of scenario1 by using method I.

TABLE 4.1
THE RESULT OF METHOD I (SCENARIO 1)

From node	To node	Travel time	Demand node	Cumulative demand	Remainder load	Number of vehicles	Vehicle utilization
1	6	12	0	0	800	1	
6	8	9	170	170	630	1	87.50%
8	9	9	300	470	330	1	
9	10	5	180	650	150	1	
10	1	21	50	700	100	1	
1	4	8	0	0	800	2	
4	2	3	120	120	680	2	
2	3	3	100	220	580	2	96.25%
3	5	8	250	470	330	2	
5	7	7	200	670	130	2	
7	1	9	100	770	30	2	
Total time		93	1470				

Table 4.1 is the result of method I used to solve the VRP problem in Table 3.1 for scenario 1. The result shows details regarding the routing from-to nodes, total travel time = 93 units' time, and the customer demands do not exceed

the vehicle capacity. The number of vehicles are equal to 2 units for which the vehicle utilization= 87.50% and 96.52%, respectively. The graphical VRP can be plotted as Figures 4.1, 4.2 and 4.3 for methods I, II and III. The results of method II and III do not show the detail like in Table 4.1 but the graphical plot in Figures 4.2 and 4.3 clearly show the routing and the number of vehicles as follows;

FIGURE 4.1
BEST ROUTE OF METHOD I (SCENARIO 1)

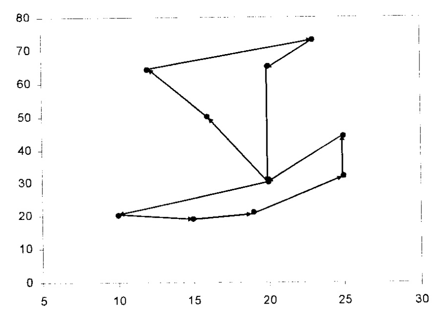


FIGURE 4.2
BEST ROUTE OF METHOD II (SCENARIO 1)

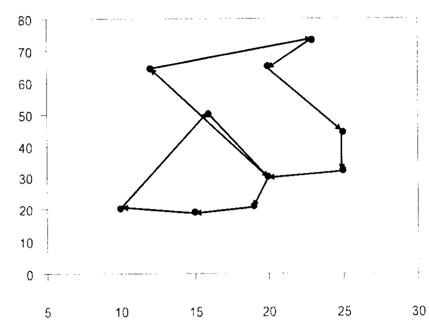
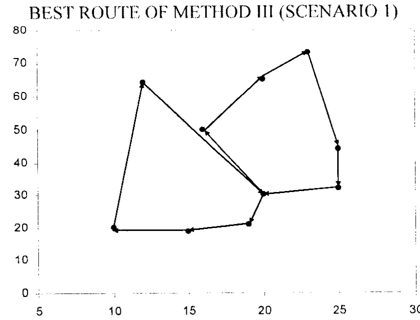


FIGURE 4.3
BEST ROUTE OF METHOD III (SCENARIO 1)



The graphical results from Figures 4.1-4.3 show different routing but the numbers of vehicles are the same at 2 units. These results are the 3rd of 9 solutions that are the candidate robust solutions. Tables 4.2-4.4 illustrate the results for all methods and scenarios as below.

TABLE 4.2
THE RESULT OF METHOD I FOR ALL SCENARIOS

Scenarios	Case	Best route	Number of Vehicles	Total travel time
1	Optimistic	Tour#1 1--->6--->8--->9--->10--->1 Tour#2 1--->4--->2--->3--->5--->7--->1	2	93
2	Most likely	Tour#1 1--->6--->8--->9--->10--->1 Tour#2 1--->4--->2--->3--->5--->7--->1	2	117
3	Worst	Tour#1 1--->6--->8--->9--->10--->1 Tour#2 1--->4--->2--->3--->5--->7--->1	2	234

TABLE 4.3
THE RESULT OF METHOD II FOR ALL SCENARIOS

Scenarios	Case	Best route	Number of Vehicles	Total travel time
1	Optimistic	Tour#1 1--->3--->2--->4--->6--->1 Tour#2 1--->8--->9--->10--->7--->5--->1	2	100
2	Most likely	Tour#1 1--->3--->2--->4--->6--->1 Tour#2 1--->8--->9--->10--->7--->5--->1	2	125
3	Worst	Tour#1 1--->3--->2--->4--->6--->1 Tour#2 1--->8--->9--->10--->7--->5--->1	2	249

TABLE 4.4
THE RESULT OF METHOD III FOR ALL SCENARIOS

Scenarios	Case	Best route	Number of Vehicles	Total travel time
1	Optimistic	Tour#1 1--->3--->2--->4--->8--->1 Tour#2 1--->6--->10--->9--->7--->5--->1	2	113
2	Most likely	Tour#1 1--->3--->2--->4--->8--->1 Tour#2 1--->6--->10--->9--->7--->5--->1	2	141
3	Worst	Tour#1 1--->3--->2--->4--->8--->1 Tour#2 1--->6--->10--->9--->7--->5--->1	2	283

From Tables 4.2-4.4, 9 candidate solutions occur during the experiment. Finally, these 9 solutions are tested by 100 scenarios; scenario S is generated randomly by uniform distribution of interval travel times. The values of $dev(X,S)$, $sse(X,S)$ and $risk(X,S)$ are calculated by equations (1), (2) and (3), respectively. The maximum values of these three robustness criteria are shown as Table 4.5.

TABLE 4.5
THE ROBUSTNESS CRITERIA RESULTS

Scenario/Criterion	Method	Method I	Method II	Method III
Scenario 1 : Optimistic case	$dev(X,S)$	93.21	99.23	106.04
	$sse(X,S)$	8687.18	9845.62	11243.76
	$risk(X,S)$	93.21	99.23	106.04
Scenario 2: Most likely case	$dev(X,S)$	69.83	74.23	78.04
	$sse(X,S)$	4876.47	5509.37	6089.71
	$risk(X,S)$	69.83	74.23	78.04
Scenario 3: Worst case	$dev(X,S)$	114.14	132.99	147.28
	$sse(X,S)$	13027.79	17686.44	21690.50
	$risk(X,S)$	0.00	0.00	0.00

Table 4.5 shows the maximum values of robustness criterion, $g_s(X)$ in equation (12). The robustness objective value, Z is found from equation (11) where $Z = \min\{h_x(S)\}$ and $h_x(S) \geq g_s(X)$. Figures 4.4-4.6 illustrate the results of these three criteria compared among the 9 candidate solutions. The minimum value of Z shows the best worst-case performance of the solutions or it is the robust solution that has potential and strength against the perturbation of traffic conditions.

FIGURE 4.4
THE ROBUST DEVIATION CRITERION COMPARISON

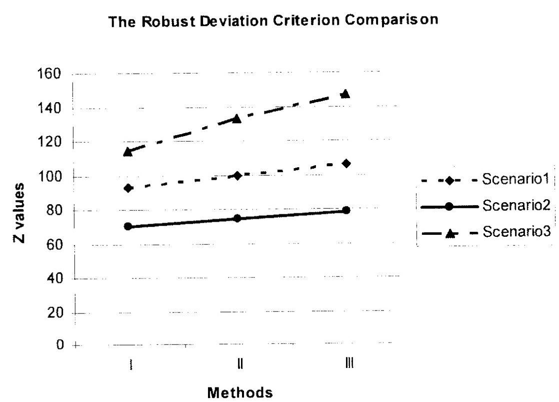


FIGURE 4.5
THE SUM SQUARE ERROR ROBUSTNESS CRITERION COMPARISON

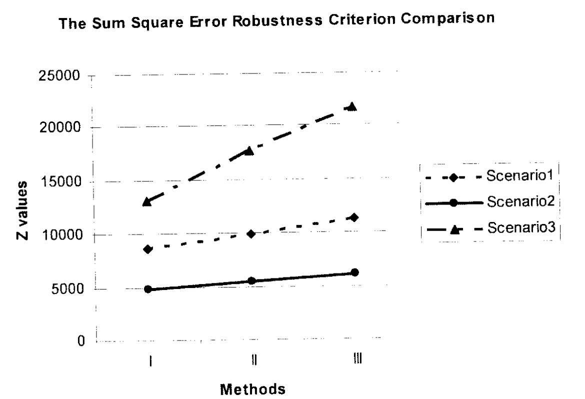
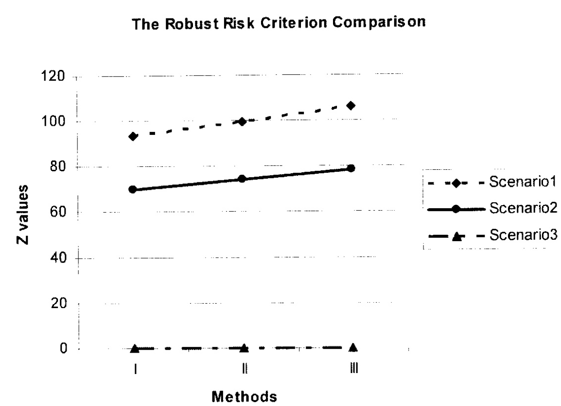


FIGURE 4.6
THE ROBUST RISK CRITERION COMPARISON



The results in Figures 4.4-4.6 are the robustness criteria comparisons that clarify the robust solution in a minimum point of individual graphs. Figures 4.4 and 4.5, the robust deviation and the sum square error robustness criteria, have the same trend in graphical direction. The robust solution for these criteria is scenario 2 by using method I. Figure 4.6 is the robust risk criterion, where the robust solution belonging to this criterion is scenario 3 with the three methods giving about the same performance.